Construct Hadronic Tensor in a Simple Way

Shu-yi Wei  (魏樹一)

September 22, 2014
The cross section for Drell-Yan process,

\[ d\sigma = \frac{\alpha^2}{s q^4} L_{\mu\nu} W_{\mu\nu} \frac{d^3 l}{l_0} \frac{d^3 l'}{l'_0} = \frac{\alpha^2}{2 s q^4} L_{\mu\nu} W_{\mu\nu} d^4 q d\Omega. \]

The hadronic tensor,

\[ W_{\mu\nu} (p_a, p_b, q, S_a, S_b) = \frac{1}{(2\pi)^4} \int d^4 \xi e^{i q \xi} \langle AB | J^\mu (0) J^\nu (\xi) | AB \rangle. \]
The cross section for Drell-Yan process,

\[ d\sigma = \frac{\alpha^2}{s q^4} L^{\mu\nu} W_{\mu\nu} \frac{d^3 l}{l_0} \frac{d^3 l'}{l'_0} = \frac{\alpha^2}{2 s q^4} L_{\mu\nu} W^{\mu\nu} d^4 q d\Omega. \]

The hadronic tensor,

\[ W^{\mu\nu}(p_a, p_b, q, S_a, S_b) = \frac{1}{(2\pi)^4} \int d^4 \xi e^{i q \xi} \langle AB | J^\mu(0) J^\nu(\xi) | AB \rangle. \]

Properties of this tensor,

- **U(1) gauge invariance.** \( q_\mu \times W^{\mu\nu} = 0. \)
- **Space-reflection Invariance.** \( W^{\mu\nu}(p, S) = W_{\mu\nu}(\tilde{p}, -\tilde{S}). \)
- **Hermiticity.** Structure functions are all REAL.
Momenta: \( p_a, p_b, q \) & Spin vectors: \( S_a, S_b \).

We switch from \( A_i \) to \( \tilde{A}_i \). Where \( \tilde{A}_i \cdot q = 0 \).

\[
\tilde{A}_i^\mu = A_i^\mu - \frac{A_i \cdot q}{q^2} q^\mu
\]

I.e., we have,

\[ W^{\mu\nu}(q, \tilde{p}_a, \tilde{S}_a, \tilde{p}_b, \tilde{S}_b) \]
\[ \epsilon^{abcd} \times \epsilon^{efgh} = g^{ae} g^{bf} g^{cg} g^{dh} - gggg + gggg - gggg \cdots \]

\[ \downarrow \]

We drop all terms that contain more than ONE \( \epsilon \)
(Pseudo) Scalars, (Axis) Vectors, Tensors

(I) Scalars

\[ S_a \cdot S_b \]
\[ \epsilon_{q\tilde{p}_a\tilde{p}_b\tilde{S}_a} \]
\[ \epsilon_{q\tilde{p}_a\tilde{p}_b\tilde{S}_b} \]
\[ \tilde{p}_a \cdot S_a\tilde{p}_a \cdot S_b \]
\[ \tilde{p}_b \cdot S_a\tilde{p}_a \cdot S_b \]
\[ \tilde{p}_a \cdot S_a\tilde{p}_b \cdot S_b \]
\[ \tilde{p}_b \cdot S_a\tilde{p}_b \cdot S_b \]

(II) Pseudo Scalars

\[ \epsilon_{p_ap_bS_aS_b} \]
\[ \epsilon_{q\tilde{p}_a\tilde{S}_a\tilde{S}_b} \]
\[ \epsilon_{p_bqS_aS_b} \]
\[ \tilde{p}_a \cdot S_b \]
\[ \tilde{p}_b \cdot S_a \]
\[ \tilde{p}_a \cdot S_a \]
\[ \tilde{p}_b \cdot S_b \]
\[ \tilde{p}_a \cdot S_b\epsilon_{q\tilde{p}_a\tilde{p}_b\tilde{S}_a} \]
\[ \tilde{p}_b \cdot S_a\epsilon_{q\tilde{p}_a\tilde{p}_b\tilde{S}_b} \]

(III) Vectors

\[ \tilde{p}_a^\mu \]
\[ \tilde{p}_b^\mu \]
\[ \epsilon_{\mu q\tilde{p}_a\tilde{S}_a} \]
\[ \epsilon_{\mu q\tilde{p}_b\tilde{S}_a} \]
\[ \epsilon_{\mu q\tilde{p}_a\tilde{S}_b} \]
\[ \epsilon_{\mu q\tilde{p}_b\tilde{S}_b} \]

(IV) Axis Vectors

\[ \tilde{S}_a^\mu \]
\[ \tilde{S}_b^\mu \]
\[ \epsilon_{\mu q\tilde{p}_a\tilde{p}_b} \]

(V) Tensor(s)

\[ d^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2 \]
One Spin Vector Case

(1) T and TS

\[ d^{\mu\nu}[q, \tilde{p}_a, \tilde{p}_b, \tilde{S}_a] \]

(2) VV and VVS

SCALARS

\[ \epsilon_{q\tilde{p}_a\tilde{p}_b\tilde{S}_a} \]

VECTORS

\[ \tilde{p}_a^\mu[\nu, q, \tilde{p}_a \tilde{S}_a] \]
\[ \tilde{p}_b^\mu[\nu, q, \tilde{p}_b \tilde{S}_a] \]
\[ \epsilon_{\mu q\tilde{p}_a\tilde{S}_a} \]
\[ \epsilon_{\mu q\tilde{p}_b\tilde{S}_a} \]

\[ \tilde{p}_a^\mu \tilde{p}_a^\nu[q, \tilde{p}_a, \tilde{p}_b, \tilde{S}_a] \]
\[ \tilde{p}_b^\mu \tilde{p}_b^\nu[q, \tilde{p}_a, \tilde{p}_b, \tilde{S}_a] \]
\[ \tilde{p}_a^\mu \tilde{p}_b^\nu[q, p_a, \tilde{p}_b, \tilde{S}_a] \]
\[ \tilde{p}_b^\mu \tilde{p}_b^\nu[q, p_a, \tilde{p}_b, \tilde{S}_a] \]
(3) AA and AAS

\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] S^\nu_a \quad [\mu, q, \tilde{p}_a, \tilde{p}_b] [\nu, q, \tilde{p}_a, \tilde{p}_b] [q, \tilde{p}_a, \tilde{p}_b, \tilde{S}_a] \]

(4) AVP

\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_a^\nu \tilde{p}_b \cdot S_a \quad [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_b^\nu \tilde{p}_b \cdot S_a \]

\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_a^\nu \tilde{p}_a \cdot S_a \quad [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_b^\nu \tilde{p}_a \cdot S_a \]
(3) AA and AAS

\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{S}_a^\nu \]

(4) AVP

\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_a^\nu \tilde{p}_b \cdot S_a \]
\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_a^\nu \tilde{p}_a \cdot S_a \]
(3) AA and AAS

\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{S}_a^\nu \]

(4) AVP

\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_a \tilde{p}_b \cdot S_a \]
\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_a \tilde{p}_a \cdot S_a \]
\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_a \tilde{p}_b \cdot S_a \]

\[ [\nu, q, \tilde{p}_a, \tilde{p}_b] \tilde{S}_a^\nu \]

\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_b \tilde{p}_b \cdot S_a \]

\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_b \tilde{p}_a \cdot S_a \]

\[ [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{p}_b \tilde{p}_a \cdot S_a \]

\[ [\mu, q, \tilde{p}_a, \tilde{p}_b, \tilde{S}_a] \tilde{p}_b^\nu = -d_{\mu
u} [q, p_a, p_b, S_a] + [\mu, q, \tilde{p}_a, \tilde{p}_b, \tilde{S}_a] \tilde{p}_a^\nu + [\mu, q, \tilde{p}_a, \tilde{p}_b] \tilde{S}_a^\nu \]

\[ \tilde{p}_{a/b} \cdot S_a [\mu, q, \tilde{p}_a, \tilde{p}_b] p_{a/b}^\nu = [q, \tilde{p}_a, \tilde{p}_b, \tilde{S}_a] \tilde{p}_{a/b}^\mu \tilde{p}_{a/b}^\nu + \tilde{p}_{a/b} \cdot \tilde{p}_a [\mu, q, \tilde{S}_a, \tilde{p}_b] \tilde{p}_{a/b}^\nu + \tilde{p}_{a/b} \cdot \tilde{p}_b [\mu, q, \tilde{S}_a, \tilde{p}_a] \tilde{p}_{a/b}^\nu \]
We have,

\[ t_{1-4}^{\mu \nu} = \epsilon^{q, \tilde{p}_a, \tilde{p}_b} S_a \left\{ d^{\mu \nu}, \tilde{p}_a^\mu \tilde{p}_a^\nu, \tilde{p}_b^\mu \tilde{p}_b^\nu, \tilde{p}_a^\mu \tilde{p}_b^\nu + \text{SYM} \right\} \]

\[ t_{5-8}^{\mu \nu} = \left\{ p_b \cdot S_a, q \cdot S_a \right\} (\epsilon^{\mu, q, p_a, p_b} \tilde{p}_a^\nu / b + \text{SYM}) \]
Two Spin Vectors

### Scalars

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_a \cdot S_b$</td>
<td>$S_a \cdot S_b$</td>
</tr>
<tr>
<td>$\epsilon q \bar{p}_a \bar{p}_b \tilde{S}_a$</td>
<td>$\epsilon q \bar{p}_a \bar{p}_b \tilde{S}_a$</td>
</tr>
<tr>
<td>$\epsilon q \bar{p}_a \bar{p}_b \tilde{S}_b$</td>
<td>$\epsilon q \bar{p}_a \bar{p}_b \tilde{S}_b$</td>
</tr>
</tbody>
</table>

### Vectors

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{p}_a^\mu$</td>
<td>$\tilde{p}_a^\mu$</td>
</tr>
<tr>
<td>$\tilde{p}_b^\mu$</td>
<td>$\tilde{p}_b^\mu$</td>
</tr>
<tr>
<td>$\epsilon_{\mu q} \bar{p}_a \tilde{S}_a$</td>
<td>$\epsilon_{\mu q} \bar{p}_a \tilde{S}_a$</td>
</tr>
<tr>
<td>$\epsilon_{\mu q} \bar{p}_b \tilde{S}_a$</td>
<td>$\epsilon_{\mu q} \bar{p}_b \tilde{S}_a$</td>
</tr>
<tr>
<td>$\epsilon_{\mu q} \bar{p}_a \tilde{S}_b$</td>
<td>$\epsilon_{\mu q} \bar{p}_a \tilde{S}_b$</td>
</tr>
<tr>
<td>$\epsilon_{\mu q} \bar{p}_b \tilde{S}_b$</td>
<td>$\epsilon_{\mu q} \bar{p}_b \tilde{S}_b$</td>
</tr>
</tbody>
</table>
Two Spin Vectors

### VV and VVS

**Scalars**

\[
S_a \cdot S_b \quad \epsilon_{q\tilde{p}_a\tilde{p}_b\tilde{S}_a} \quad \epsilon_{q\tilde{p}_a\tilde{p}_b\tilde{S}_b} \\
\tilde{p}_a \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_b \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_b \cdot S_b
\]

**Vectors**

\[
\tilde{p}_a^\mu \tilde{p}_a^\nu \times \left\{ S_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_b \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_b \cdot S_b \right\} \\
\tilde{p}_b^\mu \tilde{p}_b^\nu \times \left\{ S_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_b \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_b \cdot S_b \right\} \\
\tilde{p}_a^\mu \tilde{p}_b^\nu \times \left\{ S_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_b \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_b \cdot S_b \right\}
\]
(1) $T$ and $TS$

\[
\tilde{d}^{\mu\nu} \times \left\{ S_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_b \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_b \cdot S_b \right\}
\]

(2) $VV$ and $VVS$

\[
\tilde{p}_a \tilde{p}_a^\nu \times \left\{ S_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_b \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_b \cdot S_b \right\}
\]

\[
\tilde{p}_b \tilde{p}_b^\nu \times \left\{ S_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_b \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_b \cdot S_b \right\}
\]

\[
\tilde{p}_a \tilde{p}_b \times \left\{ S_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_a \cdot S_b \quad \tilde{p}_a \cdot S_a \tilde{p}_b \cdot S_b \quad \tilde{p}_b \cdot S_a \tilde{p}_b \cdot S_b \right\}
\]

(3) $AA$ and $AAS$

\[
\tilde{S}^\mu_a \tilde{S}^\nu_b + \text{SYM}
\]

(4) $AVP$

\[
\tilde{S}^\mu_a \tilde{p}_a \tilde{p}_a \cdot S_b \quad \tilde{S}^\mu_a \tilde{p}_a \tilde{p}_b \cdot S_b \quad \tilde{S}^\mu_a \tilde{p}_b \tilde{p}_a \cdot S_b \quad \tilde{S}^\mu_a \tilde{p}_b \tilde{p}_b \cdot S_b
\]

\[
\tilde{S}^\mu_b \tilde{p}_a \tilde{p}_a \cdot S_a \quad \tilde{S}^\mu_b \tilde{p}_a \tilde{p}_b \cdot S_a \quad \tilde{S}^\mu_b \tilde{p}_b \tilde{p}_a \cdot S_a \quad \tilde{S}^\mu_b \tilde{p}_b \tilde{p}_b \cdot S_a
\]